example AG, which do not cut DC, how far soever they may be prolonged. In passing over from the cutting lines, as AF, to the not-cutting lines, as AG, we must come upon a line AH, parallel to DC, a boundary line, upon one side of which all lines AG are such as do not meet the line DC, while upon the other side every straight line AF cuts the line DC.

The angle HAD between the parallel HA and the perpendicular AD is called the parallel angle (angle of parallelism), which we will here designate by $\Pi(p)$ for $AD=p$."

Does Lobatschewsky class his boundary line AH among the *cutting* or the *not-cutting* lines? Evidently among the cutting lines, for under Theorem 33, referring to his equation $S'=se^{-x}$, he says—"We may here remark, that $S'=0$ for $x=\infty$, hence not only does the distance between two parallels decrease (Theorem 24), but with the pralongation of the parallels towards the side of the parallelism this at last wholly vanishes."

Agreeably to this assumption of Lobatschewsky let y be the point in space at which the decreasing distance between his parallel lines AH and DC wholly vanishes. According to Euclid's postulate 2 the terminated line Dy may be extended beyond the point y. If this is not permitted, Euclid's postulate 2 would be discredited in Lobatschewsky's geometry. Assume that Euclid's postulates hold everywhere in space. On the basis of that assumption we have the authority to locate any point as z beyond y on Dy extended. Then the point z is within the angle yAE. From z draw a straight line to the point A. This must be permitted, otherwise postulate 1 would be discredited in Lobatschewsky's geometry. Since z is within the angle yAE the straight line Az must fall between Ay and AE.

But since by Lobatschewsky's hypothesis no straight line between AHy and AE can meet DC produced, the line Az must fall between Ay and AD. That is, Az must lie on both sides of Ay at the same time. Says W. K. Clifford, an enthusiastic admirer of Lobatschewsky's Imaginary Geometry—"but the way things come out of one onother is quite lovely."

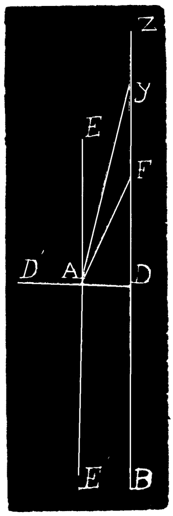


Fig. 2.

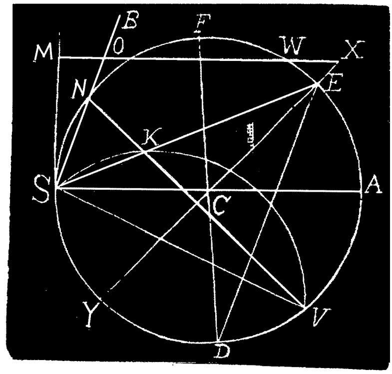
SOME FALLACIES OF AN ANGLE TRISECTOR.

By LEONARD E. DICKSON, M. A., Fellow in Mathematics, The University of Chicago.

Since 1860, Mr. L. S. Benson of New York City has labored to throw

discredit upon vital parts of mathematics. He easily trisects any angle by simple geometry and because the results following from his construction are not in harmony with results derived from Trigonometry, he quietly casts overboard as false all Trigonometry and with it the applied mathematics!

His trisection, however, really offers an interesting fallacy especially from a graphical standpoint. The *gist* of the construction given in his circulars "Mathematics out of joint," etc., is as follows:



To trisect any angle BSA between 60° and 90°, construct the right angle LSA and a circle with any point C on SA as center. Make arc $AW=60°$ and draw $WM \parallel SA$. Take $WX=MO$ and draw diameter $XECY$. Draw chord $ED \parallel SB$ and diameter DF. Then arc NA is trisected by F and E. Proof: arcs NE, SD, FA are equal, also $FE=YD$, also $NF=SY=EA$. Draw chord $DV \parallel SE$. Describe circle SKV with radius$=SC$ through points S and V. Then arc $SN=$arc SK, each being double the measure of $\angle NVS$; similarly, $\angle ESV$ makes arc $KV=$arc EV; $\angle NVE$ makes arc $KV=$arc NE. Since arcs $SKV=SDV$ and $KV=EV=SD$, then $SN=SK=DV$. $\therefore$ arc $SNE=$arc EVD and finally arc $SY=$arc YD. $\therefore$ arcs $NF=FE=EA$.

Since 3 arc $AW=180°=3$ (arc $AE+$arc EW)$=$arc $AN+3$ arc EW, we find arc $SN=3$ arc EW. Thus any angle $<90°$ is trisected.

The step used by Mr. Benson which lacks proof is the fact that NV and SE intersect on the arc drawn through S and V with radius$=SC$. It can be shown by Trigonometry that this fact is true only for one particular angle 67½°. But the error is so small that it is scarcely apparent in an ordinary-sized figure, even if drawn accurately. But with a larger figure and the favorable case of an angle near 60° or 90°, the error is distinct. The deception is increased by the fact that to a certain difference in the lengths of the arcs NF and FE corresponds a much smaller distance between K_1 (the intersection of NV and arc SV) and K_2 (the intersection of SE and arc SV). In "another proof," Mr. Benson attempts to prove that K_1 and K_2 coincide (in the point K of the figure) but lands immediately in a rediculous *argumento in circulo.*